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Reprinted from
IEEE TRANSACTIONS ON GEOSCIENCE AND REMOTE SENSING
Vol. 29, No. 1, January 1991

105-02550/

The Simulation of the SAR Image of a Ship Wake

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***Abstract*—A computer simulation of *L*-band SAR (Synthetic Aperture Radar) images of surface-ship wakes is described, and the simulation results are compared to actual SeaSAT, satellite-borne SAR imagery. The model used in the simulation accounts for both the disturbance produced in the water by the moving ship and the influence of the background sea. The radar scatter from the wake is modeled using the Bragg mechanism. A two-scale model of the sea surface is employed in which it is assumed that the small wavelength Bragg waves are modulated by the larger scale ambient sea and wake fluctuations. The larger scale fluctuations are important because they can alter the local angle of incidence and thus cause a tilt modulation in the scattering coefficient.**

Several components of the wake are modeled: The ambient waves, the Kelvin wake, and the turbulent wake. The model can produce long turbulent wakes, which may be bright or dark, depending on the geometry of the ship, radar, and wind, and it can produce Kelvin wakes.

***Keywords*—Radar, wake, turbulence.**

I. Introduction

USING a library of about 50 SeaSAT SAR wake images that had been compiled at the Defence Research Establishment Ottawa, it has been possible to make some general observations about SAR images of ship wakes. Providing that the spatial resolution is sufficiently fine in both azimuth and range (such as that obtainable in SAR), radar images of the wakes from surface ships seem to fall principally into four categories. These are: The Kelvin wake, the turbulent wake, wakes from internal waves, and narrow-V wakes. Not all types are seen at a given frequency; for example, narrow-V wakes have been seen at *L*-band frequencies, but not at *X*-band. The aim of this work is to study the production of the various types of wake images under different conditions. To accomplish this, a fairly comprehensive simulation of the SAR image of a wake has been developed. In this paper the Kelvin wake, turbulent wake, and the narrow-V wake are discussed. The turbulent and narrow-V wakes are the most common features in SeaSAT SAR images and are often seen as a dark streak behind a surface ship, stretching back for several kilometers. The Kelvin wake is seen only occasionally but has some important indirect effects on the image.

It should be noted at this point that we have expanded the term "narrow-V" wake in this paper to include occlusions of ambient waves by the turbulent hydrodynamic wake and reflections from the hull. Our formula for the boundaries of the wake arms is similar to that of Lyden [1]. However, in his case the waves are created by the ship and it is necessary to relax the Bragg condition or to postulate "random phases." It is difficult to estimate the amplitude of Lyden's waves, although they may very well be there in practice. In contrast, our waves are simply ambient waves and no relaxation of the Bragg condition is required.

The Bragg mechanism of scattering, which is assumed here, is capable of explaining most of the phenomena associated with ocean scatter, except at small radar depression angles and higher radar frequencies [2]. An improvement has been made to this model by Holliday [3], [4], which is roughly equivalent to the inclusion of the effect of varying the angle of incidence to agree with the waves slopes. It appears that this results in an enhanced-scattering cross section at *X*-band over that predicted by the simple Bragg scatter mechanism. This occurs because the longer waves make a significant contribution.

The decrease in RCS, associated with the turbulent wake astern of a ship, would seem to indicate that for this type of wake there is a reduction in the amplitudes of the surface waves responsible for the scattering of the EM waves. Despite this indication, it must be noted that sometimes the dark streak may also exhibit bright edges and perhaps other structure, neither of which are completely explained by a reduction in surface wave amplitudes.

When a vessel is underway, a "turbulent" hydrodynamic wake stretches out behind it [5], [6]. This type of wake consists not only of turbulent vortices, but it also has a steady flow component concentrated in a narrow region along the ship's track. While the theory describing the production of the turbulent wake is not sufficiently advanced to be able to make accurate predictions for a given ship, an order of magnitude estimate of the size of the various quantities can be obtained readily. It can also be shown that the nonsteady component of the vortices is not likely to be important to the radar scattering problem, and therefore the treatment of only the steady component is necessary.

Once the profile of the mean flow has been found, the effect of a wave from outside the wake impinging upon the mean flow can be studied. The theory is similar to that involved in the propagation of acoustic waves in the atmosphere in the presence of wind. The theory is well established [7]. It turns out that because the phase velocity of surface gravity waves is quite small at wavelengths corresponding to *L*- or *X*-band, the effect of a change in the flow velocity of a few centimeters per second over a distance of a few meters can have a dramatic effect. Thus a wave traveling from outside the wake will generally significantly change its wavelength, direction and phase and group velocity as it enters the wake. Indeed, for some positions and orientations within the wake it may be impossible for a wave outside the wake to propagate into it. This clearly suggests the possibility that serious reductions in scattering cross section can occur.

In the present study a simplified approach is adopted. The scattering is restricted to Bragg reflection. It is assumed that, for a small area about each position within the wake, scattering

Manuscript received October 20, 1989; revised January 19, 1990.

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IEEE Log Number 9036098.

0196-2892/91/0100-0149$01.00 © 1991 IEEE

arises from a Bragg wave. The raypath of this Bragg wave is then traced back to the outside of the wake so that the amplitude of the impinging wave can be found. A simplified power law spectrum [8] is chosen, and it is assumed that this spectrum of waves is generated by the wind at some distance from the wake and that the spectrum has a simple dependence on the wind direction. While the wind will produce new waves which decay, as described by Hughes [9], it is assumed that the wind cannot produce significant new waves over the extent of the wake itself and that the associated damping of the existing waves is negligible. This is highly desirable in that we want a steady-state situation to exist during the imaging time. These assumptions are reasonable for *L*-, but not for *X*-band. This is because the longer *L*-band Bragg waves do not damp out as quickly as the very small *X*-band waves.

To calculate the local angle of incidence of the radar waves, the simulation package combines simulated sea-height data with Kelvin wake data. This permits the variations in RCS to be found according to the "tilt modulation" mechanism. The RCS modulation produced by the turbulent wake is then added into the simulated data.

The velocity of the sea surface produces a Doppler shift which is interpreted by the radar as a change in the azimuthal position of the scatterer. This is the "velocity bunching" imaging mechanism. To include this effect in the simulation, the LOS (line-of-sight) velocities are calculated along with the sea and Kelvin wake wave heights. The LOS velocities are then used to find the appropriate azimuthal positions of surface scatters in the image plane.

The effect of speckle is also included by performing the simulation over a grid of small cells, much smaller in size than a resolution cell, and then combining complex amplitudes from these small cells coherently to form the contribution from one resolution cell. Typically, 16 subcells per resolution cell (single-look) are used in the simulation. This tends to give reasonable Rayleigh statistics for the background.

II. The Kelvin Wake

The Kelvin wake comprises the pattern of surface-gravity waves astern of a moving vessel. The waves are mainly confined to a wedge-shaped region with a half-angle of about 19.5°. Within the wedge there are two sets of waves: Divergent and transverse. The divergent waves are of shorter wavelength and their crests tend to be aligned in the general direction of the ship velocity. The longer transverse waves have crests roughly perpendicular to the ship's track. Where the crests meet at the wake edge, a caustic region occurs and cusp waves are formed.

The theory of the Kelvin wake has been studied by Havelock [10], [11] and summarized by Wehausen and Laitone [12]. Tuck *et al.* [13] have attempted to apply the theory to a simulation of the Kelvin wave heights from a moving ship. However, although the contour plots of simulated wakes produced by their method seem reasonable, the accuracy of their calculation is not adequate for our purposes. Furthermore, it does not model the caustic region.

Application of the Kelvin wake theory involves using the Green's function to describe the flow about a point source (or sink) of fluid. When combined with appropriate boundary conditions, this gives the velocity potential near the surface, and from this the wave heights and velocities can be calculated. Using the "thin ship" approximation, the ship hull is represented as a distribution of sources and sinks located on the longitudinal centerplane of the vessel, and for simple shapes the result is a single integral, which is asymptotically correct for large distances. In contrast to Tuck *et al.* [13], who employed a stationary-phase method, we evaluate it using the method of steepest descents. This gives the usual difficulty at the caustic, where amplitudes become very large. The problem at the caustic is overcome with the use of an Airy integral approximation as described in [7].

An examination of the integral shows that the shorter divergent waves tend to be produced by sources close to the water surface, and the longer transverse waves by the deeper sources. Therefore fast shallow draft vessels will exhibit a wake with strong divergent wave structure, and slow deep-draft ships will produce a wake with mainly transverse waves. Since most sources and sinks are located close to the bow and stern of a ship, much of the character of the Kelvin wake can be described by examining the interference taking place between waves created near these two positions.

Fig. 1 shows a plot of the classical Kelvin wave pattern. Fig. 2 gives an example of the cusp-region portion of a Kelvin wake. These plots were obtained by using the method of steepest descents for calculating the integral in the interior of the wake, and using the Airy integral method in the cusp region. A vertical line source (corresponding approximately to the bow) was used for plot calculation.

III. The Turbulent Wake in the Fluid

The treatment of the turbulent wake consists of a description of the turbulent wake from a towed vessel, modified by additional effects caused by the momentum production of a self-propelled ship.

The turbulent wake created by a towed vessel occurs because water, which has a small but finite viscosity, wets the submerged surface of the ship. This causes a frictional drag which imparts momentum to the fluid. The rate of transverse diffusion of this momentum by the velocity field of vortices tends to be small. No matter how the wake starts, by the time it has moved downstream it will be random [5]. Thus eddies of a wide range of scales will be produced, as well as a mean flow component directly astern of the ship along the ship track. In other words, for a towed vessel the wake can be regarded as an extension of the boundary layer.

Two main types of shear flow are treated in the literature. These are the two-dimensional flow past a long cylinder-oriented transverse to the principal flow direction, and the axisymmetric three-dimensional wake behind a body of revolution. Information concerning the mean flow velocity distributions and the development of the shape of the wake can be obtained using the mixing-length theory of Prandtl [14]. The theory is based on the idea that the appropriate dynamical quantity can diffuse throughout the flow in an analogous manner to molecules diffusing in a gas. In a sense, the "mixing length" is similar to the mean free path, but the diffusing elements are macroscopic "lumps" of fluid rather than microscopic particles.

With Prandtl's assumptions about the geometrical and mechanical similarity of the flow in different sections of the wake, the differential equations can be solved for the region of the turbulent wake located away from the object creating the disturbance. Because of the separation between the source and the disturbance, it can be expected that the solution cannot depend too greatly on the nature of the source of the disturbance itself, and that the results obtained using this solution will be more or

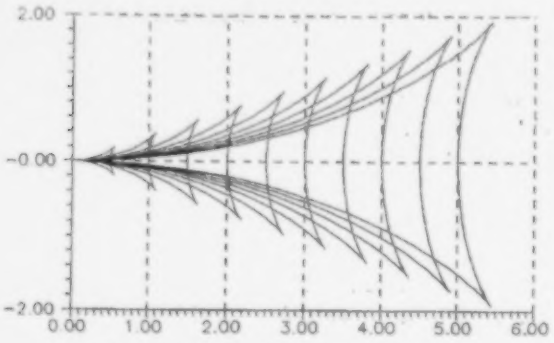

Fig. 1. Positions of Kelvin wake crests.

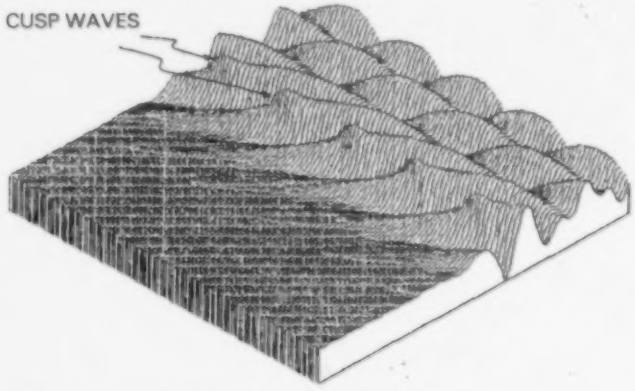

Fig. 2. An example of the cusp wave portion of the Kelvin wake.

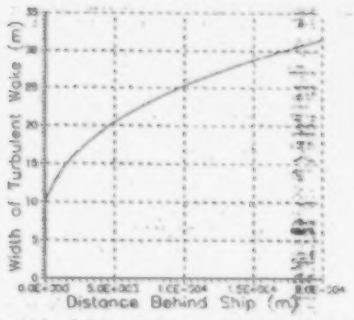

Fig. 3. The half-width of the turbulent wake behind a frigate-sized ship.

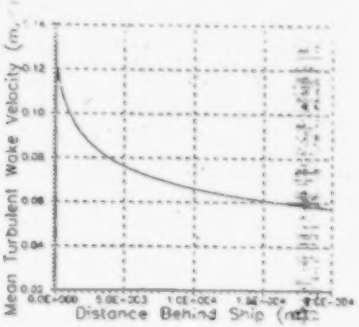

Fig. 4. The mean velocity at the center of the turbulent wake astern of a frigate-sized ship.

less generally applicable. Observations for the wake generated by a sphere have been discussed by Uberoi and Freymuth [15]. Their findings can be generalized to other than spherical bodies, as long as the appropriate drag coefficient is used to represent the rate of momentum production. Hence these results can be employed to infer the order of magnitude of the wake parameters astern of a ship.

To employ the Uberoi and Freymuth [15] method to estimate wake parameters for a ship, a drag coefficient representing the net rate of production of linear momentum by a self-propelled ship is also needed. The production of momentum by the screws will tend to cancel that caused by skin drag. However, because of wave-making resistance and wind drag forces, there will typically be a net production of fluid momentum in the opposite direction to the ship velocity. At moderate speeds the drag caused by the production of momentum by the Kelvin wake tends to be of the same order as that due to viscous drag. Since the approximate wake width and mean velocity of the flow, obtained using this theory, depend on the drag coefficient to the power of $1/3$, estimates based on the skin drag of a towed body can be utilized. Figs. 3 and 4 show graphs of the wake half-width and the velocity at the center of the wake as a function of distance behind a frigate-sized ship. A drag coefficient of 0.1 has been used in creating the graphs. It should be noted in the figures that the wake actually persists for several km. This is caused by the rather slow rate of spreading.

The screws will also contribute to the turbulent wake [16]. Their effect is to accelerate fluid located forward of the screws through the propeller disk into the propeller race, thus creating a thrust. (The thrust appears as a reaction force on the propeller blades.) This causes a reduction of pressure in the race and imparts some angular momentum as well as linear momentum to the fluid. By this means an "angular momentum wake" is formed. Numerical analyses have shown that the flow velocities and the additional diffusion caused by turbulence in this wake tend to be small.

IV. Surface Wave Propagation

In this section the effects of surface wave propagation on the wake are described. It is assumed that the surface waves propagate on a surface upon which the velocity changes smoothly from point to point. It is also assumed that the velocity changes over a distance equal to a wavelength are very small. The propagation of waves under these conditions has been treated in the literature [7], [8], [17] and will not be redeveloped here. Instead, only salient points from those previous treatments will be outlined.

If a ship is not accelerating, the frequency of a wave measured at any point in its reference frame is the same. However, the same cannot be said of the frequency measured in a frame which is stationary with respect to the local water surface. It will generally be Doppler shifted in frequency. If ω is the angular frequency outside the wake in a frame in which the water is stationary, and σ is the angular frequency measured by an observer moving with the fluid, the usual treatment of the Doppler effect gives the following relationship between σ, ω, and $\underline{U}$, where $\underline{U}$ is the local fluid velocity:

$$\sigma = \omega - \underline{k} \cdot \underline{U}. \qquad (1)$$

The notion of a local frequency σ is useful because the wavelength on the surface is directly related to it. For example, a

wave impinging on a wake flow which has a component of velocity in the same direction as the wavevector will have a reduced local frequency and therefore its wavelength will be increased. For dispersive waves, the phase and group velocities will also change.

For radar wavelengths at L-band, the Bragg waves will typically be in the gravity wave regime where the propagation is determined by the competition between inertial and gravitational forces. Because the changes in the wake velocity are small over a wavelength, ray theory can be applied. It is assumed that the changes in the mean wake velocity and wake breadth are small compared to the wake breadth, so that the water surface over the wake can be treated as a two-dimensional medium, stratified in a direction parallel to the ship's track. Refraction of rays passing through this medium can be analyzed by noting that the phase of the wave must be continuous across a line of stratification. This leads to Snell's law.

Lighthill [7] has shown that the component of the wave-action flux density, perpendicular to the lines of stratification (in this case lines of constant fluid velocity), is a constant along a raypath. The complete wave-action flux vector is defined by

$$\underline{B} = \rho g A^2 (\underline{U} + \underline{C}_g)/(2\sigma). \tag{2}$$

Here A is the wave amplitude, $\underline{C}_g$ is the local group velocity, and ρ is the density of the fluid. The use of the above equation allows the wave ampitude of a ray at various points across the wake to be calculated. In addition to dealing with the "stretching" or "contraction" of a wave as it is Doppler shifted, it takes into account the changing cross section of a "ray tube" as it passes through the medium. In this context, a "ray tube" is defined as the region enclosed by two neighboring rays. For a complete solution this equation must be augmented by the dispersion relations and the expressions for the phase and group velocities. A solution of the complete set of equations shows that some waves may be totally reflected from the wake, even for small flow velocities.

After the wave outside the wake has been specified, the above relationships allow the parameters of a gravity wave propagating into a wake to be found. For radio waves to be backscattered, the water waves must have the Bragg wavelength and be oriented parallel to the projection of the radio-wave vector on the mean water surface. In performing the simulation we want to specify the waves inside the wake, and trace rays back to the outside. This is opposite to what has just been described. For waves traveling against the flow, it may happen that, at some positions in the wake, a solution to the equations does not exist. These points in the wake will not support a Bragg wave and therefore their radar cross section will be zero.

Generally, the waves outside the wake can be described by a spectral density such as a modified Phillip's spectrum. The modification to the Phillip's spectrum is required to introduce a factor to take account of the orientation relative to the principal wind direction. Even though we do this, it may be expected that the dependence on orientation may not be all that strong. In the present study we only examine the radar cross section of two extreme cases of orientation dependence. This places bounds on the possible extremes of radar cross section. In the first case we consider a strongly anisotropic spectral density, and in the second, an isotropic spectrum. The anisotropic spectral density is given by [18]

$$P_0(k, \theta) = k^{-4} \cos^2(\theta - \alpha) \tag{3}$$

for $-\pi/2 < (\theta - \alpha) < \pi/2$, and $P_0 = 0$ otherwise. The angle α is the mean direction of the wind relative to the radar, k is the wavenumber, and θ is the wavevector angle.

A further modification is required to account for the change in the wavelength (or k) that occurs as a wave propagates through the wake. For a fixed P, if k changes, the wave-vector angle also changes. Using a Jacobean, a correction can be made for this transformation.

Programs have been written to generate the relative spectral density of the Bragg waves. Because the RCS is proportional to spectral density, these plots also show RCS. The algorithm used does the equivalent of tracing rays from a given point in the wake to the outside of the wake. Rays traveling both towards and away from the radar are examined. The final RCS is the sum of both. For each point on the wake section two tests are made. The first is to determine if a contribution to the wave amplitudes can arise from a reflection process in the wake, and the second is to establish whether it is possible for a ray to propagate from the outside of the wake into it. In some circumstances there appear to be abrupt changes in the radar cross section, and these are associated with:

(a) The onset of reflection as the observation point moves into the wake, and

(b) changes in the wave spectrum as a function of orientation.

Plots of the relative scattering amplitude for positions across the wake are shown in Figs. 5–7 for the wake produced 1-km downstream of a large merchant ship. The Bragg wavelength has been set at 0.3 m. Different values for the radar and principal wind direction for the "anisotropic" spectrum are chosen to illustrate the variety of radar signatures. Reference to the figures shows that the turbulent wake can have one of three principal forms: Dark, bright, or dark with bright edges. It can also exhibit more complicated structure. In the dark with bright-edge wake case it may be asymmetric. For example, the bright arm side of the wake is caused by the reflection of Bragg and ambient waves. Whereas on the leeward side there are no waves reflected and hence the RCS is not enhanced.

The case of an isotropic wind-generated wave spectrum is also interesting. Clearly, the wind direction is irrelevant and it turns out that the radar cross sections are independent of the sign of the angle of incidence. There are three basic signatures—all are symmetric. These are the bright wake, the dark wake, and the dark wake with bright edges. An example of the latter is shown in Fig. 8.

V. Narrow-V Wakes

There is yet another possible effect associated with the turbulent wake. It occurs when ambient gravity waves are reflected from the turbulent wake and augment the incoming ambient waves. This produces an area of brightness in the image. As the ship moves along its trajectory, the ambient waves, created at a large distance from the wake, propagate towards the turbulent wake and may be reflected in the radar-range direction to become Bragg waves. The edge of the region of excess brightness therefore propagates outwards from the wake in the range direction at the group velocity of the waves. This forms a wedge of bright return with a half-angle that depends on the ship speed. Similarly, the turbulent wake may occlude ambient waves to produce a dark wedge. This situation occurs to a small extent for the case illustrated in Fig. 7.

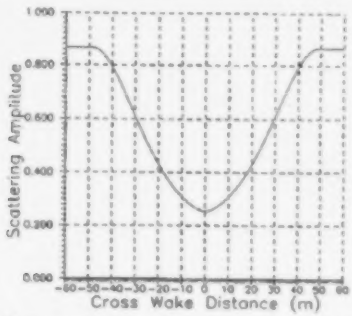

Fig. 5. The relative scattering amplitude across a dark turbulent wake.

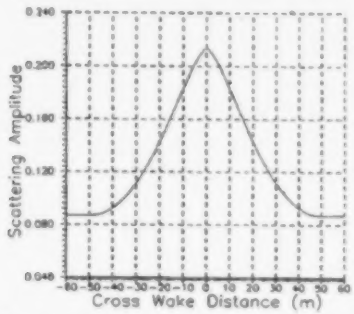

Fig. 6. The relative scattering amplitude across a bright turbulent wake.

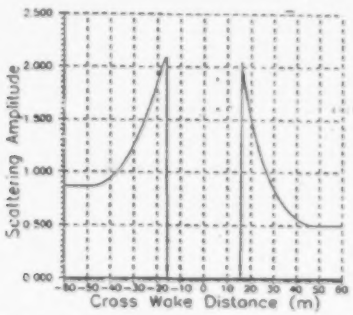

Fig. 7. The relative scattering amplitude across a dark turbulent wake with bright edges.

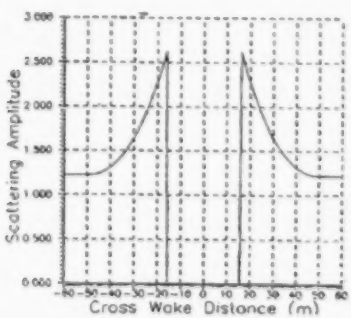

Fig. 8. The relative scattering amplitude with isotropic wind waves for a dark wake with bright edges.

These bright and dark wedges constitute a form of narrow-V wake. Their angles are typically very small. One exception is for craft moving slowly, with velocity vectors confined to a band of angles about the range direction. An example of the normal situation is as follows: If the group velocity of Bragg waves of wavelength 0.3 m is about 0.35 m/s, then a ship traveling at 10 kn (5 m/s) in the azimuthal direction, may produce a narrow-V wake with an angle of 4°.

Bright and dark lines in the image can also by produced by reflections or occulsions of ambient waves by the ship hull itself.

If the directional spectrum of ambient waves is isotropic, reflections will be compensated by occlusions and narrow-V wakes will not be formed. However, when the spectrum is anisotropic, as will most often be the case, reflections may occur from directions in which the ambient waves are weak, while strong ambient waves might be occluded. This could result in a net reduction of scatter and produce a dark region.

In Seasat imagery it is likely that many dark wakes seen are actually narrow-V wakes, rather than turbulent wakes. This conjecture is made because the width of the hydrodynamic turbulent wake tends to remain small and it is easily washed out by the velocity-bunching mechanism. Therefore, it is unlikely that many such wakes would be visible.

VI. Sea Simulation

The sea heights and velocities are simulated in a manner similar to that of Mastin *et al.* [19]. A random field of complex numbers is formed using the computer random number generator. This field is then multiplied by a two-dimensional function which represents the square root of a Pierson–Moskowitz spectrum, modified according to Hasselmann (as noted in [19]) to include a directional term. To find the velocity field, the filter function is multiplied by components of the wave-vector. The results are then Fourier transformed in two dimensions. Reference to Mastin *et al.* [19] shows that this method produces very realistic-looking seas. An example of sea heights for sea state 3, is shown in Fig. 9.

VII. Comparisons With SeaSAT Imagery

To verify the accuracy of the simulation, the simulation results were compared to actual *L*-band SAR images of 25-m resolution which were produced using data from the SeaSAT satellite-borne SAR. To generate the appropriate wake images the simulation required the specification of a large number of radar, ship, and ocean condition parameters. Many of these were unknown, but to reduce these as much as possible SeaSAT scatterometer data were employed to estimate the sea state and the wind speed and direction. A number of scenes containing wakes were compared with the output from the simulation. Fig. 10 and 11 show a real and simulated image (note that SeaSAT had a nominal range of 850 km and a nominal velocity of 7.5 km/s). The simulated image is presented with a scale that is twice that of the real image. The real image, from a ship in sea state 3, seems to include a dark narrow-V wake extending some distance astern of the ship. The wake also exhibits an area of general brightness on one side. These properties are shared by the simulated image.

Fig. 12 also shows a dark wake, which is possibly a turbulent wake oriented in the range direction. The corresponding simulation is presented in Fig. 13. It can be seen that there is good qualitative agreement between the original and simulated wake components.

A bright V-wake is shown in Fig. 14 with the simulated version in Fig. 15. Once again, there is good qualitative agree-

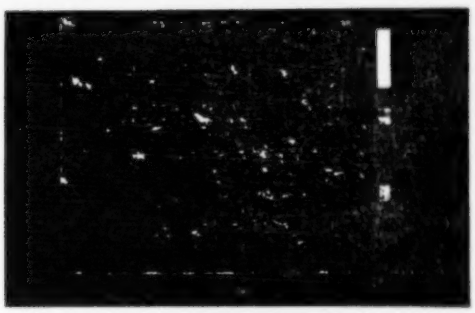

Fig. 9. A false color plot of simulated sea heights for sea state 3.

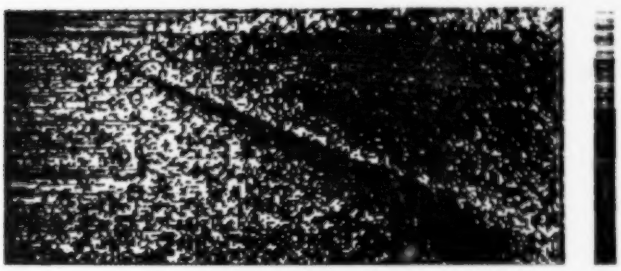

Fig. 10. A SeaSAT image of a dark narrow-V wake.

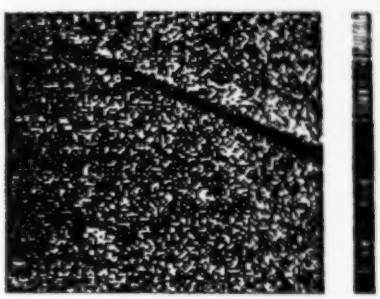

Fig. 11. A simulated version of the narrow-V wake.

Fig. 12. A SeaSAT image of a dark wake.

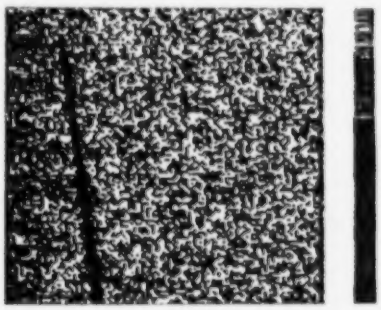

Fig. 13. A simulated version of the dark wake.

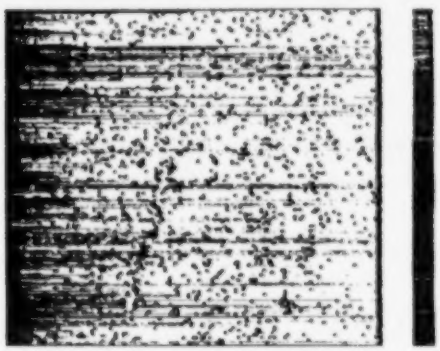

Fig. 14. A SeaSAT image of a bright wake.

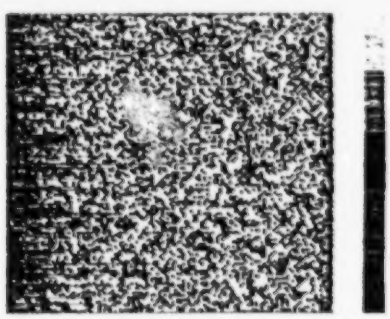

Fig. 15. A simulated version of a bright wake.

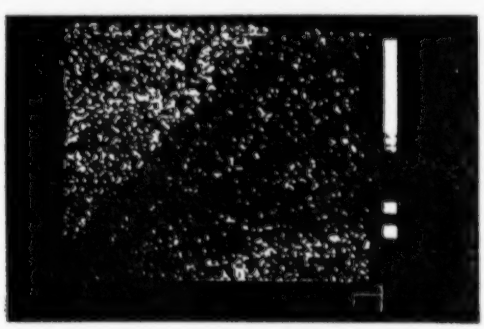

Fig. 16. Simulated image of a dark Kelvin wedge.

ment. In this case the simulation seems to indicate that one arm of the wake is a bright turbulent wake, and the other is due to reflections of ambient incident waves from the hull.

It should be noted that for Figs. 10, 12, and 14 the actual ship and wake parameters were unknown. For Fig. 11, the ship length was 200 m, ship speed was 12 kn, wind direction was 65° relative to azimuth, and sea state 3 was used. For Fig. 13, the ship length was 300 m, ship speed was 4 kn, wind direction

was 70° from azimuth, and the sea state was less than 1. For Fig. 15, the ship length was 300 m, ship speed was 3 kn, wind direction was 174° from azimuth, and sea state was again less than 1.

Most wakes can be cast into the above three categories. The characteristic Kelvin wake angle also sometimes appears, although what appears at first to be a Kelvin wake is sometimes actually a narrow-V wake. The simulations indicate that in very light seas the surface wave structure of the Kelvin wake can reduce the coherency of the signal returns within the wedge and cause a general darkening of this area in the image (Fig. 16). This darkening seems apparent in some of the real wake images.

VIII. Conclusions

The simulation which has been developed leads to results that seem to be at least qualitatively consistent with the SeaSAT data. The model certainly confirms the observation from the SeaSAT data that the turbulent (or narrow-V) wake should persist for many kilometers behind a frigate-sized or larger ship. In cases where SeaSAT scatterometer data have been used as input to the program, there is a semiquantitative agreement between the SeaSAT imagery and the simulations. Part of the reason for the agreement being only semiquantitative is that the scatterometer data are not very reliable when the sea state is small. Furthermore, a specification of the wind direction alone may be insufficient to predetermine the type of wake—it may also be necessary to know the angular dependence of the wind-generated wave spectrum.

Under the appropriate circumstances the effects of the turbulent wake can be seen, at least for large ships, even at sea states as high as 4. However, other wake components seem to require sea states less than 2 before they become visible. For instance, the Kelvin wake rarely appears directly in images. This may be partly because the wave structures tend to have a wavelength which is short compared to the resolution cell size of the 25-mm resolution SeaSAT data. But another contributing factor might be that at high sea states, the transverse waves can produce sufficient velocity bunching to wash out the potential structure in the wake.

According to the simulation, even wakes from slow ships can be imaged. Wakes from large vessels moving at 1.5 kn may be seen. However, the sea state must be small and the wind direction must be very close to the azimuthal direction for wakes to appear in the image.

Many ship wakes will not be observable because of the special geometrical conditions that must be satisfied.

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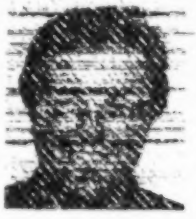

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